Modeling Polarisation Distortion of Optical Quantum Signals Through A Pressurized Window

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We study the polarization distortion of optical photons traversing an optical window under mechanical stress due to a pressure differential between both sides of the window. We confirm only a minor effect for light entering the window at normal incidences, as previously established [2]. However, we do find that for non-normal incidences the effect of birefringence on polarization encoded photon signals is noticeable and may no longer be neglected.

REAL-WORLD APPLICATION

This research was inspired by the Space QUEST mission proposal that aims to test decoherence of entangled photon pairs due to gravity [3]. The mission proposes to transmit polarization entangled optical signals from ground to a receiver telescope located inside the International Space Station (ISS) (Fig. 1). As the quantum signal beam passes through a pressurized window, it will experience birefringence [4].

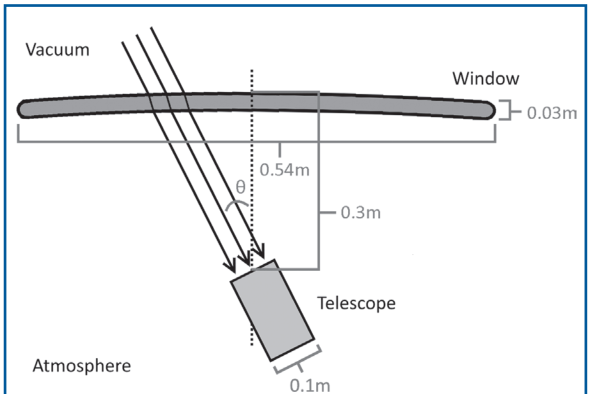

Fig. 1 Diagram of the signal beam that would reach the telescope through the window. The telescope is located inside the pressurized crew compartment.

FINITE ELEMENT MODELING

The window's birefringence is modeled using finite element modeling (FEM). Each finite element has a different stress tensor and consequently, birefringence [5-7]. We implemented a ray-tracing algorithm that calculates the particular birefringence caused by the individual stress-elements each ray passes. The polarization distortion caused by each optical element of the FEM grid is represented by an individual Jones matrix [8]. The total Jones matrix (J_T) experienced by a particular ray is determined by accounting for the Jones matrices of all the elements it transverses. By combining all the rays across the aperture of the telescope, the overall polarization distortion induced by the window is estimated.

Outline of the Polarization Analysis Algorithm

The quantum optical signals are considered in two different polarization bases. Horizontal (***H***) and vertical (***V***) polarization states define the HV-basis, while diagonal (***D***) and antidiagonal (***A***) polarization states define the DA-basis. We determine the polarization distortion measured as the Quantum Bit Error Ratio (QBER) for each ray, defined as:

$$\text{QBER} = (\boldsymbol{V}'\,J_T\boldsymbol{H} + \boldsymbol{H}'\mathbf{J}_T\boldsymbol{V} + \boldsymbol{A}'J_T\boldsymbol{D} + \boldsymbol{D}'J_T\boldsymbol{A})/4 \qquad (1)$$

The QBER effectively measures the probability that a photon would switch polarization from H to V, or from D to A, or vice versa, upon passing through an element.

The stress in the window is defined as a 3D mesh of cubic element of the window (201×201×40 cubes), each with a corresponding stress tensor represented as:

$$\sigma = [\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy}]. \qquad (2)$$

Stress (σ_{PS}) in the principle strain axis of each element is a diagonal matrix of the eigenvalues of the element's stress matrix (σ). The columns of the matrix V correspond to the eigenvectors, such that the following equation holds.

$$\sigma * V = V * \sigma_{PS} \qquad (3)$$

Summary

This article discusses a method for modeling the polarization distortion of a quantum optical beam passing through a pressurized window at arbitrary angles.

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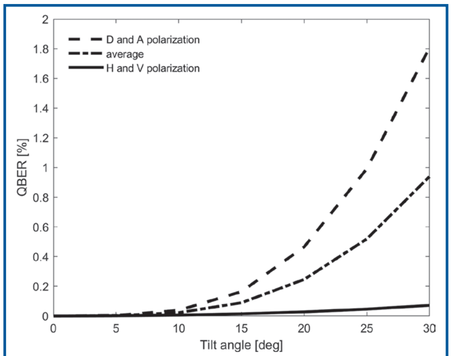

Fig. 2 Overall QBER of the entire beam entering the telescope, as a function of incoming beam tilt angle (0 deg = normal incidence).

The birefringence in the principle strain axis is calculated by multiplying the material's dielectric impermeability tensor (q) by σ_{PS} [9]. The V matrix rotates it back to the window's frame, and R_θ is applied to rotate it according to the tilt of the incoming ray (θ), resulting with the birefringence (ΔB) defined in the rays' reference frame.

$$\Delta B = R_\theta \cdot V \cdot q \cdot \sigma_{PS} \cdot V' \cdot R'_\theta \tag{4}$$

ΔB is then rotated about the ray's axis by angle γ, such that the birefringence (ΔB_P) is in its own principal reference frame, to simplify the Jones matrix representation.

$$\gamma = \frac{1}{2}\tan^{-1}\left(\frac{2\Delta B_{xy}}{\Delta B_{xx} - \Delta B_{yy}}\right) \tag{5}$$

$$\Delta B_P = R_{\gamma,3D} \cdot \Delta B \cdot R'_{\gamma,3D} \tag{6}$$

With ΔB_P, we can calculate the change in refractive index orthogonal components of the polarization would experience giving Δn_x and Δn_y [10].

$$\Delta n_x = -1/2\, n_0^3 \Delta B_{P,x} \quad \Delta n_y = -1/2\, n_0^3 \Delta B_{P,y} \tag{7}$$

The changes in refractive index cause phase shifts (δ_x, δ_y) in their respective components. The wavelength of light in the window medium is λ_m and the distance it travels in the element is L.

$$\delta_x = 2\pi\Delta n_x L/\lambda_m \quad \delta_x = 2\pi\Delta n_y L/\lambda_m \tag{8}$$

These phase shifts result in a polarization distortion (J_P) in the principal axis. The matrix $R'_{\gamma,2D}$ rotates it back to the beam's reference frame to determine the distortion of one element (J_e).

$$J_{principal} = \begin{bmatrix} e^{i\delta_x} & 0 \\ 0 & e^{i\delta_y} \end{bmatrix} \tag{9}$$

$$J_e = R'_{\gamma,2D} \cdot J_{principal} \cdot R_{\gamma,2D} \tag{10}$$

By accounting for the distortions of all the elements a ray passes through, we can determine the total polarization distortion (J_T) that each single ray experiences.

$$J_T = J_e \ldots \cdot J_3 \cdot J_2 \cdot J_1 \tag{11}$$

RESULTS AND CONCLUSIONS

We calculated the birefringence for an incoming beam at various angles of tilt, θ, with the vertical polarization defined to be in the plane of incidence. The overall QBER of the entire beam is effectively negligible for zero tilt (normal incidence), in agreement with previous work [2]. However, as the incidence angle increases, the impact of mechanical stress causes a significant QBER as seen in Fig. 2. In future work, we plan to model various system parameters such as beam aperture, and study methods to reduce or compensate for the QBER using birefringent elements. The authors would like to thank NSERC, CFI, ORF, and CSA for their funding and support.

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