# SPINS FIRST? WAVE-FUNCTIONS FIRST? QUBITS FIRST!

**SUMMARY**: I discuss strengths and shortcomings of the “waves first” and “spins first” strategies for teaching introductory quantum mechanics and advocate for what I call “qubits first”.

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Quantum Mechanics is taught in two or three courses in typical physics programs, often starting with a “Modern Physics” course in second year that presents historical experiments before introducing wave mechanics. Students find it interesting, instructors enjoy teaching it, researchers apply (more advanced) quantum mechanics skillfully and successfully. But in the end, “what it all means” remains mysterious to many physicists at all levels, despite them being fully comfortable with “the math” [1].

Why is that? Part of the reason is certainly that quantum phenomena are detached from everyday experience. Second-year students have just learned how to use an abstract language to express concepts much closer to their experience, such as forces, momentum, or energy, but are probably not ready for the jump in abstraction required to think about wave functions as probability amplitudes. Finding suitable in-class demos is also a challenge, and the introductory quantum course may look like an exercise in solving differential equations to many students.

But I believe the more important reason - and one that we should address now that we celebrate quantum mechanics’ approximately 100$^{th}$ birthday in this International Year of Quantum Science and Technology - is the sequence of content that is typically chosen for quantum courses. The usual approach is to present historical experimental insights starting from the Bohr model (quantization of energy levels), the photoelectric effect (photons as energy packets) and the Compton effect (photons have momentum, emphasizing particle-wave duality), and then follow the historical timeline by introducing the wave function to describe simple systems. Some textbooks, including the widely used one written by Griffiths, even start with the Schrödinger equation and wave function without any prior motivation. This approach requires the students to accept new, abstract and counterintuitive concepts that are typically not stated explicitly (the waves-first approach often does not introduce the postulates at the beginning) while discussing physical variables (position and momentum) that are deceptively close to classical ones. At the same time, students must apply relatively advanced mathematical tools that they have typically just learned, such as the theory of ordinary differential equations. This curriculum increases the cognitive load unnecessarily. Not only does this approach make it harder for the students to learn quantum mechanics, but it also blurs the distinction between actual quantum mysteries and confusion due to the mathematical description. Even worse, this approach obfuscates core quantum concepts such as uncertainty: a student who learns to "explain" position-momentum uncertainty in terms of Fourier analysis will likely view quantum uncertainty as a side-effect of the mathematical description instead of a fundamental feature of quantum systems, including discrete ones.

For other physics courses, we have long abandoned the historical approach: for example, we do not teach Newtonian mechanics using geometry. Instead, we simplify it using "modern" mathematics, in the form of calculus. We also use a more intentional pedagogical approach using simplified models: we start introducing mechanics with point masses instead of rigid bodies, and electricity and magnetism with point charges before introducing electric fields - so why should we introduce the wave function first instead of using a simple two-level system (which we can call a qubit to illustrate its generic nature and its use in current technical applications)?

One reason is probably tradition, combined with reluctance to change something that appears to be working fine if one doesn't look too closely. Another reason might be a desire to avoid thinking about "philosophy" (more on that below). While these sentiments should not be strong reasons to avoid updating the curriculum, there is a real obstacle: workload. Fundamentally rethinking how we teach quantum mechanics means redesigning not only the introductory course, but also adjusting the whole sequence of quantum courses, with possible effects on other parts of the program. This can affect many instructors in a department, who may simply not have the bandwidth to deal with such changes. Fortunately, the redesign of any course after the introductory one requires essentially only a rearrangement of the order in which topics are taught, and clarifying the learning outcomes.

Given these nontrivial obstacles, is it worth the effort? Student difficulties with traditionally taught quantum mechanics are well documented in the Physics Education Research (PER) literature [2], and reducing the extraneous cognitive load for our students is certainly a worthy goal. Starting with two-level systems like spin-1/2 particles allows them to focus on understanding the new concepts while using very simple linear algebra. Introducing the postulates of quantum mechanics with two-level systems requires only that students think of two-dimensional (complex) vectors and their representations in different bases, 2x2 matrices, and probabilities, as opposed to infinite-dimensional Hilbert spaces, square-integrable functions, and probability densities. When students learn about wave mechanics later, they can identify the wave function as the position representation of an object they are familiar with, the ket. This connection represents a crucial insight that often gets lost in the waves-first approach.

In addition to reducing the extraneous cognitive load, the spins-first approach gets to the core of quantum concepts like states, measurement, probabilistic outcomes and uncertainty right away. Starting from the experimental results of a Stern-Gerlach experiment, it emphasizes the fact that physics is an experimental science (which sometimes gets lost for students struggling with what they see as "just math"), and that the probabilistic nature of quantum mechanics is a fact of life. In the waves-first approach, this probabilistic nature is much less clear because it is often introduced as "smearing out" the location of a particle, which can appear as artificially imposing fuzziness on a perfectly adequate classical variable. Using a spin-1/2 system also introduces uncertainty as a fundamental feature of quantum mechanics based on experimental observations of e.g. two spin components, as opposed to using the Fourier transform relating position and momentum.

For some instructors, switching to a spins-first approach might feel uncomfortable because it implies taking a more deliberate stance on what many would dismiss as "philosophy", the so-called interpretations of quantum mechanics. Describing a spin-1/2 system with a two-dimensional vector requires introducing the concept of the state of a system as a container for all we can know about it.

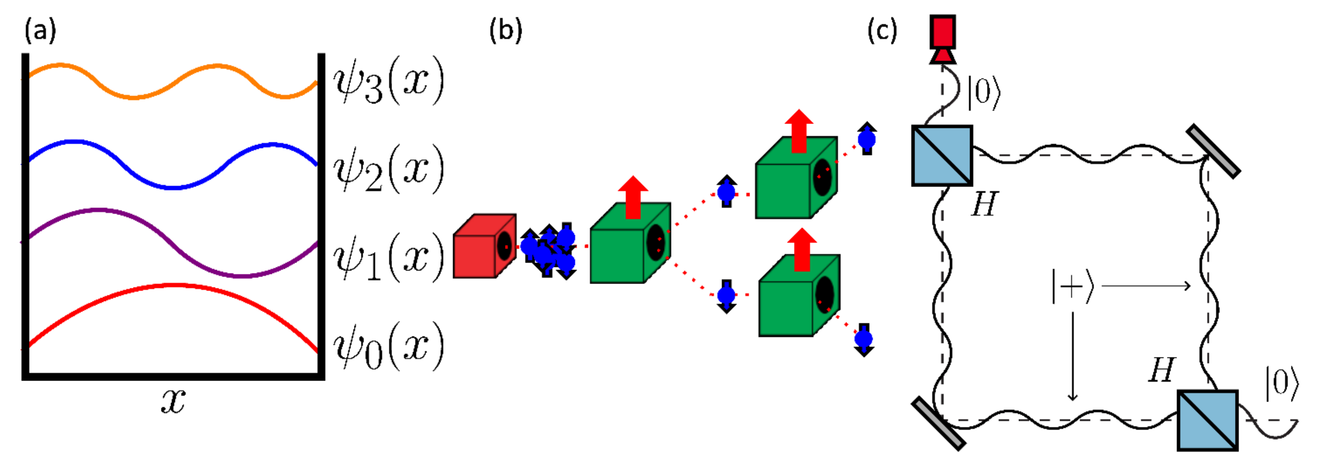


Figure 1. These images show the model systems we use for teaching quantum mechanics: The bound states in a potential well (a) are typical for the waves-first approach. They don't show the physical system, just the mathematical solution which reminds the students of standing waves on a string - good for illustrating quantization, but bad for interpreting the amplitude. The Stern-Gerlach experiment (b) is at the core of the spins-first approach. While the image is a very simplified presentation of the experiment, it does relate to physics the students have seen before (charged particles in a magnetic field) and explicitly shows the quantization of the new (to them) quantity spin. The Mach-Zehnder Interferometer (c) plays a crucial role for the qubits-first approach. Similar to the Stern-Gerlach sketch, it shows a simplified representation of physics familiar to the students (lasers, interferometry), and a simple description using 2x2 matrices will allow them to obtain the correct result. It can also be used to illustrate the use and meaning of different bases.

This is not too far from the original Copenhagen interpretation, which treats physics epistemologically, as theory of what we can say about nature, as opposed to an ontological theory of what nature is. (Note that this is very different from the popular "shut up and calculate" that is often presented as Copenhagen interpretation). Anton Zeilinger beautifully clarified this epistemological view even further in terms of information:

> *"If we accept that the quantum state is no more than a representation of the information we have, then the spontaneous change of the state upon observation, the so-called collapse or reduction of the wave packet, is just a very natural consequence of the fact that, upon observation, our information changes and therefore we have to change our representation of the information, that is, the quantum state."* [3]

The waves-first approach does not require this epistemological clarification of what a state vector is, although it could be done. I believe that the origin of much confusion when students learn quantum mechanics is the combination of a naive "shut up and calculate" interpretation with an emphasis on variables that are familiar from classical physics, such as position and momentum. In contrast, it is much easier to proceed from the general concept of a state vector (as is done in the spins-first approach) to a specific representation (the wave function) than backwards. It is also much easier to go from 2 dimensions to infinitely many.

There are more advantages to teaching quantum mechanics in this order: Decoupling the quantum concepts from more advanced mathematical formalism frees up class time to address exciting aspects of quantum mechanics that students likely have encountered already on YouTube or social media - but doing it properly. One only needs two-level systems to illustrate entanglement. One can even introduce density matrices and discuss decoherence, which - although it does not solve the measurement problem - provides deeper insight into the quantum-classical transition. Learning these concepts early provides a big advantage in upper-level courses.

Starting with two-level systems also provides an opportunity to talk about modern developments in quantum technology, including quantum communication and quantum computing. These are fascinating topics that engage students and provide opportunities to learn about current research, a rare opportunity in undergraduate courses. In the quantum computing context, students can practice their coding skills and might even get an opportunity to program a real quantum computer (or a simulation).

The spins-first approach (see [4] for textbooks) provides many advantages over the waves-first approach - but my own favorite approach is actually "qubits first", motivated by the textbook by Pieter Kok [5]. Using a Mach-Zehnder interferometer as the first model system, one can introduce two-level systems (with a basis of paths, not polarizations) and call them qubits to illustrate that they can be described with states in an abstract space that is fundamentally different from 3-dimensional physical space. The interferometer can easily be described with vectors, matrices and probabilities, but still feels somewhat familiar to students because they have seen lasers before (as opposed to spin-1/2 particles). Even the transition from classical interference with a laser to quantum interference with a single-photon source is sufficiently intuitive for the students, although one needs to rely on simulations to "show" results.

Is this approach working? PER on the topic is only starting (see [6]), with a major hurdle being the scarcity of research tools and the difficulty of comparing student populations from different institutions. We have implemented the qubits-first approach in our introductory quantum course at SFU since 2022, with very positive feedback from our students [7], and we are continually updating and evaluating. Introducing topics like quantum information and quantum technology allows us to include hands-on demos such as those from the Quantum Explorations Student Toolbox (QuEST) kits developed by the Institute for Quantum Computing (IQC) at the University of Waterloo [8]. Some of our students have even started developing their own demos.

The spins-first (or qubits-first) approach does not try to play down or hide the quantum mysteries that make the topic challenging – on the contrary, it places them at the core of teaching and learning. By decoupling them from advanced math and historical confusion, it allows the students to focus on the fundamental quantum concepts and "what it all means". Building on these solid foundations allows the students to dig deeper into more advanced and mathematically challenging topics in upper-level courses, but they don't have to wait until then to learn about entanglement, quantum teleportation, and other exciting topics from current quantum information research.

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