CREATING A SUPERFLUID CRYSTAL IN HELIUM-3

SUMMARY: When confined to a thickness smaller than one-hundredth of a human hair, superfluid helium-3 forms a strange new state of matter; a superfluid crystal.

By **PRAMODH SENARATH YAPA**[1] <senarath@ualberta.ca>, **RUFUS BOYACK**[1,2] <boyack@ualberta.ca> and **JOSEPH MACIEJKO**[1,2] <maciejko@ualberta.ca>

[1]Department of Physics, University of Alberta, Edmonton, Alberta T6G 2E1, Canada
[2]Theoretical Physics Institute, University of Alberta, Edmonton, Alberta T6G 2E1, Canada

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THE LIQUID TO SOLID PHASE TRANSITION

In a seminal paper published in 1937 [1], the great Soviet physicist Lev Davidovich Landau decried the scientific discourse surrounding the liquid-solid phase transition. He wrote: "One even finds the strange statement that there is no essential difference at all between liquids and crystals, and that continuous transitions between them are possible. However, liquids differ essentially from crystals in that they are isotropic in contrast to anisotropic crystals. Every transition from a crystal to a liquid or to a crystal of a different symmetry is associated with the disappearance or appearance of some elements of symmetry." With these fateful words, Landau set the stage for our modern classification of phase transitions as symmetry-breaking processes. However, Landau probably would not have imagined that this same classification could lead to a state of matter which blurs the liquid-solid distinction: a superfluid crystal.

SUPERFLUID HELIUM-3

The superfluid behaviour of helium-3 was discovered in 1972 by David Lee, Doug Osheroff and Bob Richardson, for which they later won the Nobel Prize in Physics. Cooling the helium-3 down to a temperature roughly two thousandth of a degree above absolute zero, they discovered that the liquid suddenly began to flow without any friction. This was many decades after the same phenomenon was discovered in helium-4, the more common isotope of helium. This delay was because helium-3 is a fermion, and must first form a Cooper pair to transition into its superfluid phase. The Cooper pairing in helium-3 results in a much more fragile state than that of bosonic helium-4, and thus helium-3 requires lower temperatures for superfluidity.

The Cooper pairing also means that there can be many different phases of superfluid helium-3; in the language of Landau, the symmetry group of helium-3 is larger than helium-4, and can be broken into many distinct sub-groups. In the laboratory setting, it was established that the phase diagram of bulk superfluid helium-3 contained only two thermodynamically stable phases – the A and the B phases.

But as experimental techniques and nanotechnology became more sophisticated, physicists began to wonder about the confined phases of superfluid helium-3: can you make new phases by restricting the geometry of the container in which the superfluid lives?

STRIPES OR POLKA DOTS?

In 2007, a theory paper by Anton Vorontsov and Jim Sauls [2] considered this exact situation: confining the superfluid into a slab. The effect of this confinement is that the top and bottom surfaces interfere with the Cooper pairs, leading to reduced superfluidity. Vorontsov and Sauls found that this interference could be counteracted by the superfluid forming a periodic density variation along the plane. As it is the density of the Cooper pairs that is varying, this is referred to as a pair-density wave (PDW) state. At a small enough slab thickness, they predicted that the stable state of the superfluid would be a PDW varying along one direction, while remaining homogeneous in the other in-plane direction. Due to this pattern of density variation, it was dubbed a ‘stripe phase’. This stripe phase and other superfluid PDW states are unusual as they are zero-viscosity liquids, but also show the periodic density modulations which we expect to see in solids - a superfluid crystal!

In 2019 and 2020, two experimental groups tested this theoretical prediction. The first, a collaboration between Royal Holloway University of London and Cornell University [3], put the superfluid into a 1.1 µm slab and found an experimental signature of a PDW phase. But what they found was inconsistent with a stripe phase; the PDW had to vary along both in-plane directions, making it a two-dimensional PDW. Instead of a ‘stripe phase’, they saw something like a ‘polka dot’ phase.

In 2020, the experimental group led by Prof. John Davis at the University of Alberta performed a series of experiments [4] mapping out the entire temperature-pressure phase diagram for three different slab thicknesses between 600 nm and 1.1 µm. Their phase diagrams showed that this new phase was stable in all of the slabs, and appeared sandwiched between the A and B phases.

So by early 2020, experiments had made clear that it is a two-dimensional (2D) pair density wave state which shows up in a confined geometry. But then this begged a couple of questions from the theory side: why is this 2D pair-density wave stable? And what is the actual crystal structure of this pair density wave? Is it a polka dot, which has a triangular symmetry? Or would it have the symmetry of a square lattice?

A TRIANGULAR SUPERFLUID CRYSTAL

These questions about the stability and structure of the PDW were what we tackled in in our recent work [5]. We did this using an old idea proposed by Landau called ‘weak crystallization’, which relies on Ginzburg-Landau (GL) theory to construct a theory of the crystallization transition. GL theory is based on a quantity called the order parameter and in helium-3, the order parameter is a 3 × 3 matrix:

$$A_{\mu j} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

To understand this superfluid order parameter, we need to understand that helium-3 is a spin-triplet, *p*-wave superfluid. This means that the spin and the orbital angular momentum of the Cooper pairs are both equal to 1: $s = 1$, $l = 1$. This means that the Cooper pairs have 3 sub-states for each projection of the spin and orbital angular momentum. We need separate order parameters for each of these sub-states, and thus there are 3 × 3 or 9 independent complex-valued order parameters for the superfluid helium-3. The μ indices in the $A_{\mu j}$ matrix are for the spin, and the j for the orbital angular momentum. To look for a PDW phase, we ask if it is energetically favourable for this order parameter to be modulated spatially.

We first calculated which combinations of order parameter components (known as ’collective modes’) would energetically favour a periodic density variation. The energy cost for each mode should vary with the wavelength of the variation, and we can produce a plot like Figure 1 which plots the energy cost for each mode along the y-axis and the wavevector along the x-axis.

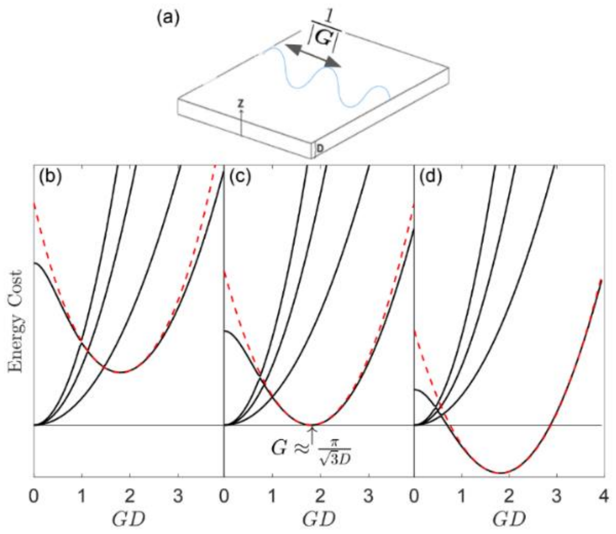

Figure 1. (a) Schematic depiction of the slab and the wavevector of the density variation, G. (b) – (d) show how a mode softens at a finite value of the wavevector as we reduce the thickness of the slab.

This theory also let us predict the crystal structure of this PDW phase - it must have the symmetries of a triangular lattice, also known as the D_6 symmetry group. The symmetry is triangular because of a specific (cubic) term in the Ginzburg-Landau free energy which forces the basis vectors of the PDW’s reciprocal lattice to add up to zero in triangles.

RESULTS

Shown in Figure 2 is our computed phase diagram for a slab thickness D = 300 nm. The blue region corresponds to the B phase under confinement, the planar-distorted B (PDB) phase; the red region corresponds to the A phase, which is unmodified from its bulk order parameter structure; and the grey region corresponds to the new PDW phase. We also show the data points for the calculated critical temperatures, T_1* and T_2* where the mode softens, and T_{PDW}, the temperature where the phase transition occurs. T_{PDW} is very close to the mode softening temperatures and is thus indistinguishable from them on the plot. Qualitatively similar results are obtained for the thicknesses studied in experiment, but the PDW region is smaller.

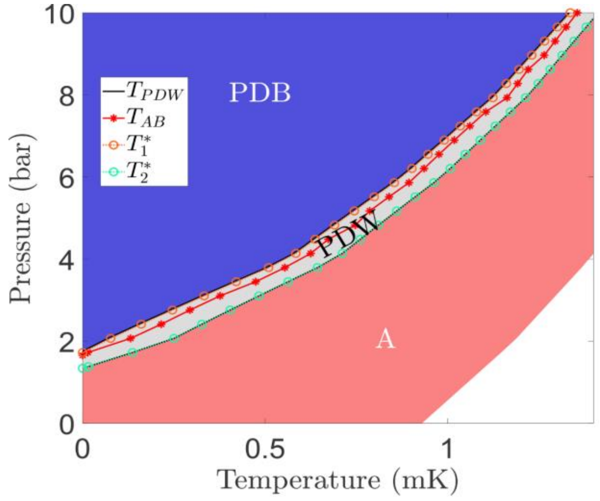

Figure 2. Calculated phase diagram of superfluid helium-3 when confined to a slab thickness D = 300nm.

For the crystal structure of this PDW, as the order parameter is a 3 × 3 matrix, we were able to compute the structure as it would appear in each of those 9 components. But of these 9 components, we found that there were three dominant terms, each with a different spatial structure as shown in Figure 3.

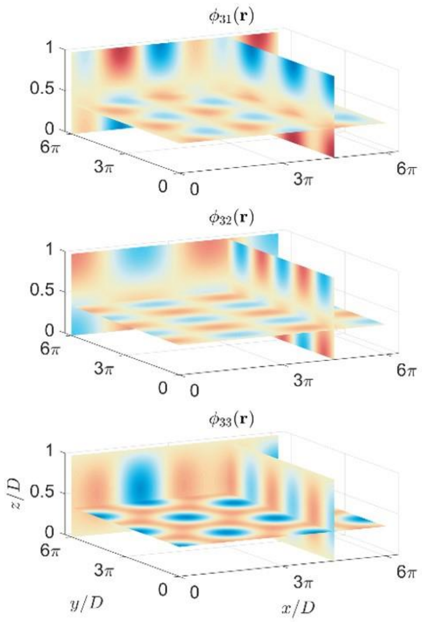

Figure 3. The three dominant superfluid crystal structures. These appear in the bottom row of the order parameter matrix ($\mu = 3$) with $A\mu j = A\mu j + \varphi\mu j$. Here $A\mu j$ is the background homogeneous superfluid order parameter.

CONCLUSION

In summary, we have proposed a mechanism which explains why a two-dimensional pair-density wave state appears in confined superfluid helium-3. This mechanism allows us to compute the phase diagram under confinement, which qualitatively matches experimental results. We are also able to predict the dominant crystal structures of this emergent superfluid crystal using this theory. In the future, we hope to calculate the properties of this state and the experimental signatures of its crystal structure.

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