DECOHERENCE RESULTING FROM THE GRAVITATIONAL INTERACTION BETWEEN TWO QUANTUM OBJECTS

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All quantum systems continuously interact with their environments and are fundamentally inseparable from them. As a result of this interaction, information about the system is irreversibly lost to the environment, which can cause the coherence (i.e., the 'quantumness') of the system to decrease over time. This process is known as decoherence.

While the phenomenon of decoherence is essential in understanding the quantum-to-classical transition, this phenomenon only became a mainstream subject of study in the 1980s [1, 2]. Still an active area of research however, decoherence is the process that a general quantum system undergoes when the quantum property of superposition in a particular basis is suppressed. As a result of interacting with its environment, a quantum system initially prepared in a spatial superposition may decohere into a mixed state, and the observability of the superposition, say in an interference experiment, will decrease over time.

Recent experimental progress has given rise to the possibility of performing experiments involving massive quantum systems in superposition. Specifically, in the field of cavity optomechanics it has been suggested that massive levitated nano-particles may be prepared in macroscopically distinct spatial superpositions [3]. Such advances have the potential to test different models of gravitationally induced decoherence [4-7].

In this article, we model these nano-particles as Gaussian wave packets and superpositions of Gaussian wave packets. We consider two of these particles interacting with each other under their own gravitational influence. Treating one of the particles as the environment, we study the decoherence process of the other particle which is initially prepared in a spatial superposition. We observe that over time this spatial superposition is suppressed and, as a measure of non-classicality, we compute the purity of this particle and observe its decrease in time.

SUMMARY

Quantum particles are modelled as Gaussian wave packets and the decoherence process resulting from the gravitational interaction between two such wave packets is studied.

GAUSSIAN WAVE PACKETS AND 1-DIMENSIONAL GRAVITY

We consider two quantum particles of equal mass separated by a distance R in 1 dimension. The first particle begins in a spatial superposition of two Gaussian wave packets, the size of the superposition being Δ. The second particle begins in a Gaussian wave packet and acts as the environment seen by the first particle. This setup is depicted in Fig. 1. Note that R and Δ are measured in units corresponding to the width of the wave packets, which in the cases considered are identical.

The composite system of the two particles evolves under the influence of their own gravitational interaction. This evolution is described by the Hamiltonian

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + Gm_1 m_2 |x_2 - x_1|, \qquad (1)$$

where G is Newton's constant in 1-dimension and m_1 and m_2 are the masses of the two particles and are chosen to be equal. The first two terms appearing in the Hamiltonian above describe the free evolution of the particles and the last term describes their gravitational interaction[2]. This evolution is simulated in Mathematica following the methods outlined in [8].

Denoting the initial state of the composite system of the two particles as $\rho_1(0) \otimes \rho2(0)$, the reduced state of particle 1 at a later time t is given by

$$\rho_1(t) = \mathrm{tr}_2\, [U(t)\rho_1(0) \otimes \rho_2(0) U(t)^\dagger], \qquad (2)$$

where $U(t) := e^{-iHt}$ is generated by the Hamiltonian given in Eq. (1). The magnitude of the matrix elements in the position basis $|\langle x|\rho_1(t)|x'\rangle|$ are plotted in Fig. 2 at four different times; the diagonal elements lie along the line $x = x'$ and the off diagonal elements lie along the line $x = -x'$. As

1. Jack Davis tied for 2nd place in the CAP Best Student Poster Presentation competition at the 2017 CAP Congress at Queen's University in Kingston, ON.
2. Recall that in D spatial dimensions the Newtonian gravitational potential is proportional to $|x2 - x1|^{2-D}$. Alternatively, the 1-dimensional case considered in Eq. (1) may be thought of as two infinite parallel plates interacting in 3-dimensions.

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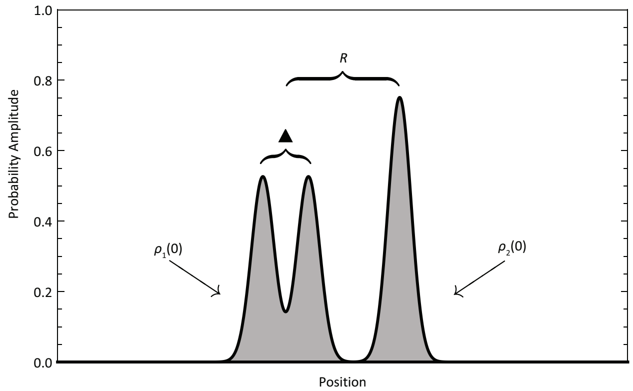

Fig. 1 Initial spatial wave functions of the two interacting particles.

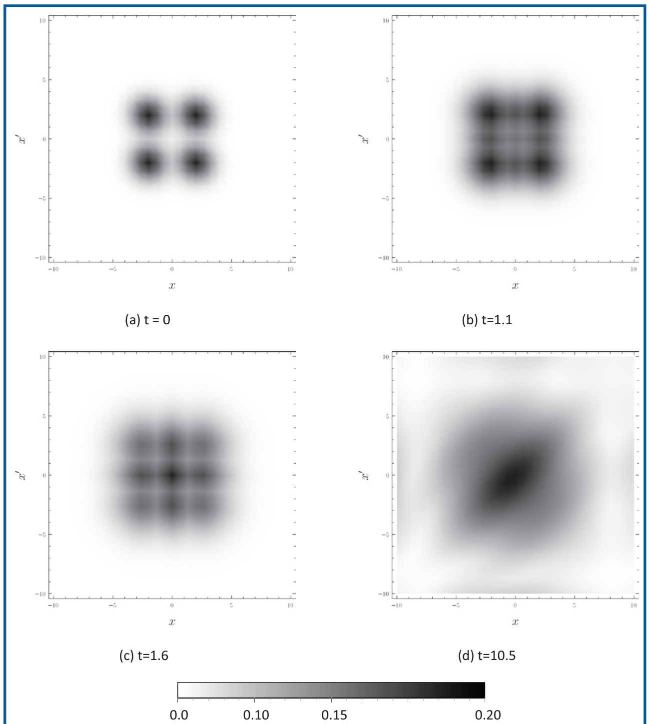

Fig. 2 The magnitude of the reduced density matrix in the position basis $|\langle x|\rho_1(t)|x'\rangle|$ is plotted at four different times. The separation of the particles is $R = 10$ and the size of the spatial superposition is $\Delta = 4$.

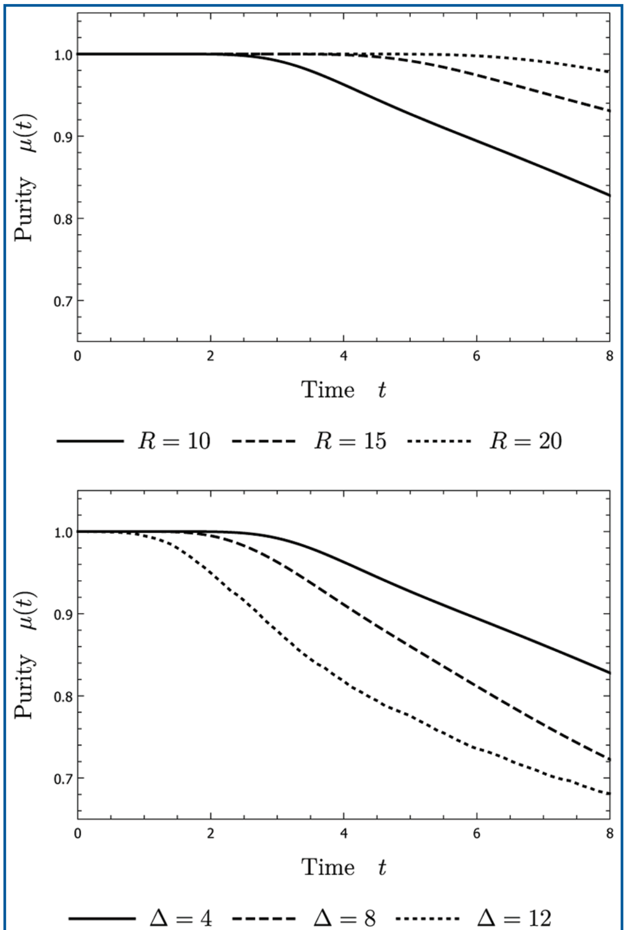

Fig. 3 The purity $\mu(t)$ is plotted as function of time for (a) different particle separations R and (b) for different sizes of superposition Δ. The solid line in both (a) and (b) corresponds to the values $R = 10$ and $\Delta = 4$.

time progresses we see that the off diagonal elements are suppressed and thus observe that the gravitational interaction between the two particles causes spatial superposition of particle 1 to decohere.

The evolution described by the Hamiltonian in Eq. (1) entangles the two particles. As a result, when the second particle acting as the environment is traced out, the reduced state of particle 1 given in Eq. (2) is mixed. To quantify how mixed particle 1 is we compute the purity associated with $\rho_1(t)$; the purity is defined as $\mu(t) := \mathrm{tr}\,[\rho_1(t)^2]$. In our particular case, $\mu(t)$ takes values between 1 (pure state) and 0 (completely mixed state). The purity is an indicator of the non-classicality of the state of a system [2], and its decrease is a signature of the onset of decoherence.

In Fig. 3 we plot the purity of particle 1 as a function of time for several sizes of superposition Δ and separation R of the two particles; by construction the purity of particle 1 at $t = 0$ is $\mu(0) = 1$. In all cases we observe that the purity decreases over time. We find the closer the particles are (smaller R) and the larger the size of the superposition of particle 1 (bigger Δ), the quicker the purity of particle 1 decreases.

It is interesting to note that classically in 1-dimension, particles moving under their mutual gravitational interaction only care about the orientation of the particles with respect to one another and not their separation[3]. However, as we observe in Fig. 3, quantum properties like the purity care about both the distance the particles are away from each other and the quantum states (i.e., superposition) of the particles.

CONCLUSIONS

In summary, we have begun investigating how quantum properties are affected by the Newtonian gravitational interaction between quantum particles with an eye on realizing these effects in near future experiments. Future work will include exploring other experimentally accessible measures of non-classicality, generalizing the above analysis to 3-dimensional Newtonian gravity, and modelling corrections to the decoherence process predicted by alternative collapse theories [9].

3. For the case at hand, the gravitational force exerted by either particle on the other is

$$F := -Gm_1m_2\nabla|x_2 - x_1| = \mathrm{sgn}\,(x_2 - x_1)Gm_1\,m_2,$$

which is independent of the particle separation.

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