COSMIC MNEMONICS OR NUMERICAL COSMOLOGY

BY DOUGLAS SCOTT, ALI NARIMANI, AND DON N. PAGE







Douglas Scott <dscott@phas. ubc.ca > and Ali Narimani, Department of Physics & Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1

Don N. Page, Department of Physics, University of Alberta, Edmonton, AB, Canada T6G 2E1 strophysicists are currently drowning in unprecedented amounts of data, including some that can be used to pin down the parameters describing the statistical properties of the entire observable large-scale Universe within the context of a simple model. As a result of these data, many scientists are hailing this as the 'era of precision cosmology'^[1].

This precision has taken another step forward with the recent publication of cosmological results from the cosmic microwave background (CMB) satellite $Planck^{[2, 3]}$. The *Planck* findings further underscore our rather full accounting of the cosmic energy budget and an assessment of how fast the Universe is expanding, as well as other quantities describing the density perturbations laid down at early times that grew into today's astronomical structures.

An often-stated result, forming the focus of the main cosmological parameter paper from *Planck*, is that *merely six* numbers are sufficient to parameterise the 'Standard Model of Cosmology' (SMC, see Refs. [4, 5] and reviews in Ref. [6]). However, the significance of this tour de force of modern physics is undermined by the difficulty of describing these parameters to a non-specialist – the Universe on the largest observable scales is fully characterised by the values of $\Omega_{\rm b}h^2$, $\Omega_{\rm c}h^2$, θ_* , $A_{\rm s}$, *n* and

SUMMARY

Our current description of the large-scale Universe is now known with a precision undreamt of a generation ago. Within the simple standard cosmological model only six basic parameters are required. The usual parameter set includes quantities most directly probed by the cosmic microwave background, but the nature of these quantities is somewhat esoteric. However, many more numbers can be derived that quantify various aspects of our Universe. Using constraints from the *Planck* satellite, in combination with other data sets, we explore several such quantities, highlighting some specific examples.

 τ (presented in Table 1 and described below), all of which need considerable explanation. Moreover, the statement that the set contains only six parameters is a little misleading for several reasons. First of all, there are other parameters that are fixed to their default values within the SMC. These include the overall curvature of space, the required mass of additional species such as neutrinos, whether the dark energy density evolves, and the existence of other types of fluctuations in the early Universe. Secondly, several parameters are determined by astrophysical measurements other than CMB temperature anisotropies. These include the overall temperature of the CMB today, the abundance of light elements such as helium, and the numbers that describe the whole of the rest of physics! And thirdly, although six parameters may be sufficient within the SMC, the choice of which parameters to include in that set is not unique. Plenty of interesting numbers can be derived from those most naturally measured quantities. A good example is the age of the Universe, which is not directly determined from CMB measurements but is easy to calculate once the SMC parameters have been pinned down.

It is surprising that the Universe can be boiled down to just half a dozen numbers, given the huge amount of cosmological information available from the CMB, as well as from galaxy surveys and other astrophysical probes. This dramatic compression of information requires that the distribution of temperature anisotropies has close to Gaussian statistics ^[15] in order for maps to be fully described by power spectra. In addition, the simplicity of the underlying physics^[16] leads to the power spectra demonstrating a vastly reduced number of degrees of freedom compared with what one could imagine. In a way, the large-scale and early Universe is quite simple, being essentially uniform, with small amplitude perturbations that are maximally random, i.e. with no correlated phases. This means that the early perturbations have none of the non-linear complexity required to describe today's small-scale objects such as galaxies, planets and people.

Since the Universe is uncomplicated enough (at least in a statistical sense) to be encapsulated in a few numerical factoids, then the simplest such quantities should be much more familiar. Every educated human should know some of the numbers that describe their Cosmos, at least as well

as the names of the local Solar System planets, and other facts, such as the dates of famous historical events, or the statistics of a favourite sports team.

Innumerable quantities could be used to articulate our present understanding of the Universe, and different cosmologists have their own favourites. Here, we select a few derived cosmic numbers, and explain how modern precision cosmology affects different ways of characterizing them.

Several quantities are easier for the non-expert to grasp, compared to the standard set. Others involve exploiting particular numerological coincidences – but we do not claim any special significance to those numbers we choose to highlight. Nevertheless, we hope that some of these quantities may help you remember your cosmic serial numbers, and grasp more fully the extent of our present understanding of the Universe in which we live.

COSMOLOGICAL DATA

We use data constraints provided by the *Planck* satellite^[2], which maps the pattern of temperature variations on the microwave sky. Such CMB experiments probe the structure of the Universe at the time when photons last interacted with matter significantly, the so-called 'last-scattering surface' about 370,000 years after the Big Bang. The power spectrum (or equivalently the correlation function) of these variations encodes information about the initial nature of the density perturbations and how they have evolved over cosmic times. Hence by measuring them accurately, we can derive the parameters that describe the large-scale Universe. Previous CMB measurements, including from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite^[17], showed that a fairly simple model, the SMC (also called 'A cold dark matter' or ACDM), fits the data and requires just six free parameters. Planck has confirmed with greater precision that this basic model still works well.

Table 1 lists the set of six parameters most directly measurable from the CMB. The 6-parameter model requires a fixed framework, including a set of testable assumptions (presented in Table 2).

There are many more things to measure about the Universe than the CMB, but it provides a high-fidelity and wellunderstood data set that is very powerful in combination with other kinds of data. Following the Planck Collaboration we elect to use the constraints coming from the *Planck* data combined with: large-angle polarisation measurements from $WMAP^{[7]}$; small scale (i.e. high multipole ℓ) CMB data from the ACT ^[8] and SPT ^[9] experiments; and a set of estimates of the so-called 'baryon acoustic oscillations' [10–14] in the relatively nearby Universe. Together, this data combination is described by the labels *Planck* +WP + HighL + BAO and gives highly precise and self-consistent determinations for the

TABLE 1

6-PARAMETER SET DESCRIBING THE BASIC COSMOLOGY, DERIVED FROM *PLANCK*^[2] PLUS OTHER DATA SETS^[7-14]. Here the Ω s represent the density as a fraction of the critical density (which makes the spatially geometry flat) and *h* is the Hubble constant in units of 100 km s⁻¹ Mpc⁻¹. See Ref.[3] for further details and coventions.

Standard cosmological parameters		
Parameter	Description	Value
$\Omega_{\rm b}h^2$	Baryon density	0.0221 ± 0.0002
$\Omega_{\rm c}h^2$	Cold dark matter density	0.1187 ± 0.0017
θ_*	Acoustic angular scale	0.010415 ± 0.000006
$A_{\rm s}$	Amplitude of density perturbations	$(2.20 \pm 0.06) \times 10^{-9}$
п	Logarithmic slope of perturbations	0.961 ± 0.005
τ	Optical depth due to reionisation	0.092 ± 0.013

cosmological parameters. Other combinations of data could be chosen, which would make only slight differences in the numerical values (some examples are shown in Fig. 1).

Reference [3] describes how a Monte Carlo Markov chain approach is used to fit cosmological models to the data, and hence to extract parameter values and uncertainties. These publicly available chains allow calculation of probability distributions for any derived quantity, and the determination of the most likely values and uncertainties; these Markov chains are provided through the Planck Legacy Archive¹. From the full distributions for any specific quantity, we present the mean and standard deviation (σ). Since most parameters are detected with high significance, the distributions are fairly bellshaped (see Fig. 1), indicating a reasonable characterisation of the constraints.

COSMOLOGICAL QUANTITIES

We will now discuss each derived quantity in turn. The precision may be of greater interest in some cases than others, and so we use the following notation: '=' means 'essentially identical', i.e., within about 1σ (note that this is empirical equivalence, and not the same thing as mathematical equivalence); ' \simeq ' means 'pretty close', i.e., within 3σ or so; and

^{1.} See http://pla.esac.esa.int/pla/aio/planckProducts.html.

TABLE 2 Basic assumptions for the 'Standard Model

OF COSMOLOGY'.

Note that all of these are testable, and have successfully passed the tests to date. Because of the dominance of dark matter (which is mostly 'cold', CDM) and dark energy (usually identified with the cosmological constant, Λ), the SMC is often referred to as the ' Λ CDM' model.

	Assumptions underlying the SMC
1.	Physics is the same throughout the observable Universe.
2.	General Relativity is an adequate description of gravity.
3.	On large scales the Universe is statistically the same
	everywhere.
4.	The Universe was once much hotter and denser and has
	been expanding.
5.	There are five basic cosmological constituents:
	5a Dark energy behaves just like the energy density of
	the vacuum.
	5h Dark matter is pressureless (for the purposes of

- 5b Dark matter is pressureless (for the purposes of forming structure).
- 5c Regular atomic matter behaves just like it does on Earth.
- 5d Photons from the CMB permeate all of space.
- 5e Neutrinos are effectively massless (again for structure formation).
- 6. The overall curvature of space is flat.
- 7. Variations in density were laid down everywhere at early times, proportionally in all constituents.

'~' means 'roughly', i.e., similar in magnitude, but not necessarily within a few σ .

Age of the Universe

Probably the conceptually simplest quantity is the age of the Universe, t_0 . In the usual units $t_0 = (13.80 \pm 0.04)$ Gyr, corresponding to 0.435 exaseconds (in S.I. units) or $\simeq 5$ trillion days. Using the fine-structure constant ($\alpha \equiv e^2/4\pi\epsilon_0\hbar c \sim 1/137$, a dimensionless number that gives the strength of electromagnetism) then $t_0 \simeq 10^8/\alpha$ years.

The Earth and the rest of the Solar System formed approximately 4.6 Gyr ago, e.g., Ref. [18] gives a precise age of (4.5682 \pm 0.0003) Gyr. This is essentially $t_0/3$ ago, so that Earth formed when the Universe was 2/3 of its present age.

Other ways of telling the time

An important parameter that describes our cosmological location is the epoch, within the evolving model, at which we are making our observations. This epoch can be defined in different ways. The obvious way is to give the value of t_0 . However, we can equivalently give the value of any of the time-evolving parameters, for example the temperature of the CMB today, which is $T_0 = (2.7255 \pm 0.0006) \text{ K}^{[19]}$.



Fig. 1 Example of probability distributions, here for the age of the Universe. The dashed line shows the results directly obtained from the Planck chains, which is well described by a Gaussian distribution, as indicated by the solid curve. This plot is specifically for the data combination coming from the 2013 release from Planck, together with large-angle polarisation data from WMAP ('WP'), additional constraints at large multipoles from SPT and ACT ('HighL') and constraints on the scale of the acoustic oscillations in the baryons at relatively low redshift ('BAO'). The other two Gaussians show how different data combinations can give somewhat different (although still statistically consistent) results.

Imagine a hypothetical situation in which we are communicating with another 'universe' where the physical constants might be different – then we would need to describe the epoch in dimensionless units^[20]. For example, the CMB temperature can be expressed dimensionlessly as a fraction of the electron mass, $\Theta = kT_0/m_ec^2 \simeq 4.6 \times 10^{-10} \simeq 2^{-31} \simeq \alpha^4/(2\pi)$, or $2.5 \times 10^{-13} \sim e^{-29}$ in terms of the proton mass.

We can also give our cosmic observational time by quoting the value of some parameters at a fiducial epoch. For example, the period when the matter and radiation density had the same value, called 'matter-radiation equality', corresponds to redshift $z_{eq} = 3410 \pm 40$ (and this would be 1.69 times higher if we compared matter with photons only). This means that length scales at the equality epoch were about 3400 times smaller than they are today in the expanding Universe, and the CMB temperature was then 9300 K, as hot as an A-type star. The age at that epoch was $t_{eq} = 51100 \pm 1200$ years. And at that epoch the Universe was expanding much faster than today, actually $H_{eq} = (10.6 \pm 0.2) \,\mathrm{km \, s^{-1} \, pc^{-1}}$ (note this is per 'pc', not 'Mpc'). Another special epoch is when the CMB photons last significantly interacted with matter, which is usually referred to as the epoch of 'last-scattering'. This corresponds to a redshift of $z_{\rm ls} = 1089.3 \pm 0.4$ and a time of $t_{\rm ls} = (372.8 \pm 1.5)$ kyr. At that epoch the CMB temperature was close to 3000 K, the surface temperature of an M-type red dwarf star.

Alternatively, the formation of the Earth occurred at a time corresponding to redshift $z_{\oplus} = 0.420 \pm 0.005$, when the CMB temperature was (3.869 \pm 0.013) K. For an observer present when Earth formed, today's epoch would be in the far future, and would correspond to $z = -0.30 \pm 0.04$.

Expansion rate

In the expanding Universe, the 'scale factor', a(t), describes how length scales evolve with time. The time derivative of the logarithm of this function *evaluated today* is known as the Hubble constant, i.e. $H_0 \equiv (\dot{a}/a)|_{t_0}$ (where a dot denotes a time derivative and a = 1 today by convention). Since it measures the recession speed per unit distance, the value of H_0 is usually given in units of km s⁻¹ Mpc⁻¹, which is dimensionally the same as the reciprocal of a time. The value is $H_0 = (67.8 \pm 0.8) \,\mathrm{km \, s^{-1} \, Mpc^{-1}or}$ (2.20 ± 0.02) × 10⁻¹⁸ s⁻¹ in inverse time units. In fact $H_0 \sim 1/t_0$, and observations were precisely consistent with that value several years ago^[21].

However, the current value is slightly less than $1/t_0$, as $H_0t_0 = 0.957 \pm 0.009$. The Hubble *parameter*, on the other hand, evolves with time: $H(t) \equiv \dot{a}/a$. Since $H_0t_0 < 1$ today, but the Hubble parameter tends to a finite value in the future while *t* increases without limit, then there must be a time in the future when Ht = 1 exactly. Let us call this special epoch the 'Milne time', t_M (since in the empty universe proposed by E.A. Milne^[22] *t* is always 1/H). It will occur (1.1 ± 0.2) Gyr from now, i.e. when the Universe is about 15 Gyr old.

At the formation time of the Earth, corresponding to $z_{\oplus} = 0.42$, the Hubble parameter was $(85.0 \pm 0.7) \,\mathrm{km}\,\mathrm{s}^{-1}\,\mathrm{Mpc}^{-1}$. In the SMC, the Hubble parameter will approach a value of $H_{\infty} \equiv H_0 \times \Omega_{\Lambda}^{1/2}$ in the far future. This quantity is independent of the observer epoch and hence is, in some sense, more fundamental than the Hubble 'constant' today. Its value is $H_{\infty} = (56.4 \pm 1.1) \,\mathrm{km}\,\mathrm{s}^{-1}\,\mathrm{Mpc}^{-1} = \sqrt{10/3}$ attohertz. This means that $1/H_{\infty} = (17.3 \pm 0.3)$ Gyr and $H_{\infty}t_0 = 0.796 \pm 0.013$.

Deceleration, jerk, snap, crackle and pop

The Hubble parameter is defined as $H(t) \equiv (\dot{a}/a)$, with the Hubble constant being the value today of the slope of the logarithm of the scale factor, $\ln a(t)$, specifically $H \equiv (\dot{a}/a)|_{t=t_0}$ (since a = 1 today). Dimensionless parameters can be defined to describe higher-order derivatives of a(t), namely: deceleration, $q_0 \equiv -(a\ddot{a}/\dot{a}^2)_{t=t_0}$; jerk, $j_0 \equiv (a^2\ddot{a}/\dot{a}^3)_{t=t_0}$; snap, $s_0 \equiv (a^3\ddot{a}/\dot{a}^4)_{t=t_0}$; crackle, $c_0 \equiv (a^4\ddot{a}'/\dot{a}^5)_{t=t_0}$ and pop, $p_0 \equiv (a^5\ddot{a}/\dot{a}^6)_{t=t_0}$ (note that these

are not unique definitions for dimensionless jerk etc., but the ones discussed here are the most common in cosmology). Fitting these quantities (now using a model that includes curvature as a free parameter), we find: $q_0 = -0.537 \pm 0.016$; $j_0 = 1.000 \pm 0.003$; $s_0 = -0.39 \pm 0.05$; $c_0 = 3.22 \pm 0.12$; and $p_0 = -11.5 \pm 0.7$.

The dominance of matter makes the Universe decelerate at early times, and dark energy drives the more recent accelerated expansion. The cross-over occurred when the deceleration was equal to zero, i.e., q = 0, which occurred at $z_q = 0.649 \pm 0.027$. This is somewhat earlier than the epoch when $\Omega_m = \Omega_{\Lambda}$, which occurred at $z_{\Lambda} = 0.31 \pm 0.02$ (and note that those epochs cannot be coincident if the dark energy behaves exactly like a cosmological constant). It may be interesting to note that the formation of the Earth (at z = 0.42) is bracketed by these two epochs, specifically about a billion years before dark energy dominated the cosmological energy budget, and a billion and a half years after the Universe started to accelerate.

Constituents

The census of the contents of the Universe is usually described in terms of the contribution to the average energy density, as a fraction of $\rho_{\rm crit}$, which is the critical value that makes space curvature flat. So we have $\Omega_{\rm b} =
ho_{\rm b}/
ho_{\rm crit}$ (= 0.0482 \pm 0.0009) for the baryon abundance, $\Omega_{\rm c} = \rho_{\rm c}/\rho_{\rm crit}$ (0.260 \pm 0.010) for (cold) dark matter, $\Omega_{\rm m}~\equiv~\Omega_{\rm b}~+~\Omega_{\rm c}~(=~0.308~\pm~0.010)$ for the total matter content and $\Omega_{\Lambda} = \rho_{\Lambda}/\rho_{\rm crit}$ $(=0.692 \pm 0.010)$ for the cosmological constant or 'dark energy'. Based on the current estimate for H_0 , we find $\rho_{\rm crit} \equiv 3H_0^2/8\pi G = (8.6 \pm 0.2) \times 10^{-27} \, {\rm kg \, m^{-3}}.$ This value is equivalent to the mass of about 5 protons or neutrons per cubic metre of space (imagine an atom of mass number 5 in each m^{-3} , easy to remember, since there *are* no nuclei of mass number 5 which are even remotely stable). In contrast, the abundance of baryons corresponds to approximately one in every sphere of 1m radius.

The cosmological constant is usually written as Λ and is the same quantity that appears in Einstein's field equations. It has units of inverse seconds squared and is related to the equivalent mass density in this component through $\Lambda = 8\pi G \rho_{\Lambda}$. The data give $\Lambda = (1.00 \pm 0.04) \times 10^{-35} \, {\rm s}^{-2}$. It can be expressed in SI units using only three words: 'ten square attohertz'. It can also be written as $\Lambda = (10.0 \, {\rm Gyr})^{-2}$. In everyday units one can express the equivalent vacuum mass density as $\rho_{\Lambda} = (6.0 \pm 0.2) \times 10^{-27} \, {\rm kg \, m^{-3}$. Since $p = -\rho c^2$ for vacuum energy, then the pressure is -5.4×10^{-10} Pa or -4.0×10^{-12} Torr, or -5.3×10^{-15} atmospheres.

The values of $\Omega_{\rm b}h^2$ and $\Omega_{\rm c}h^2$ are conventional parameters, given in Table 1. In S.I. units we have $\rho_{\rm b}=(4.16\pm0.05)\times10^{-28}\,\rm kg\,m^{-3},~\rho_{\rm c}=(2.23\pm0.03)\times10^{-27}\,\rm kg\,m^{-3},~and~\rho_{\rm m}=(2.65\pm0.04)\times10^{-27}\,\rm kg\,m^{-3}.$ One can easily define ratios of the $\Omega s,~e.g.,~\Omega_{\Lambda}$ / $\Omega_{\rm m}=2.25\pm0.11$ and $\Omega_{\rm m}$ / $\Omega_{\rm b}=6.39\pm10^{-25}\,\rm m$

0.11. It may be interesting to note that Ω_c / $\Omega_b=2\Omega_\Lambda$ / Ω_c (=5.36).

For the relativistic particle content $\Omega_r = (9.0 \pm 0.2) \times 10^{-5}$ today (including 3 species of neutrinos that for this purpose are assumed to be massless), or $\Omega_{\gamma} = (5.38 \pm 0.12) \times 10^{-5} = \alpha^2$ (for photons only).

The baryon-to-photon ratio, defined conventionally through $n_{\rm b}/n_{\gamma} \equiv \eta \equiv \eta_{10} \times 10^{-10}$ is given by $\eta_{10} = 6.13 \pm 0.08 \simeq 2\pi$ (with helium abundance being a free parameter in this particular calculation).

Initial conditions

So far, all the quantities describe a perfectly smooth Universe. However, we know there are imperfections in this picture, density irregularities laid down at early times that grew through gravitational instability into the rich structure seen today. There are several ways to parameterise the amplitude of the initial perturbations, with the conventional way being through the amplitude of the power spectrum of the Fourier modes. For example, the *Planck* team give $A_s = (22.0 \pm 0.6) \times 10^{-10}$ (actually they use log A_s) at a scale specified by wavenumber k = 0.05 Mpc⁻¹.

As an alternative, one can consider the 'lumpiness' of the density field directly. This is often expressed as the standard deviation of the variations in density, i.e., the square root of the variance σ_R^2 of $\delta \rho / \rho$, in spheres of a given radius, R. A conventional choice is to use a radius of 8 h^{-1} Mpc; this gives $\sigma_8 = 0.826 \pm 0.012$, where the h^{-1} scaling is a remnant from a time when the Hubble constant was very poorly known. Instead of using the somewhat obscure σ_8 parameter, one could instead ask for the size of sphere for which the variance is precisely unity – this turns out to be $R_{\sigma=1} = (8.9 \pm 0.3)$ Mpc (and note the lack of h scaling here).

Another way to define the amplitude would be to take the value of the density perturbation at the Hubble scale (defined explicitly through k = aH) at some special epoch, say the Milne time $t_{\rm M}$. This gives $\sigma_{\rm M} = (5.6 \pm 0.3) \times 10^{-6} \ (= 3^{-11})$, which could be considered a more observer-independent measure of the fluctuation amplitude.

In the simplest pictures for these density perturbations, they would be laid down in a way that is democratic with respect to scale – the so-called Harrison-Zeldovich initial conditions. This corresponds to a logarithmic variation of power with scale, denoted by 'n' (i.e., $n \equiv d \ln P(k)/d \ln k$) with the default value (as originally suggested by Harrison and Zeldovich) being unity. In fact, there seems to be a little more power on large scales compared to small scales, such that $n = 0.961 \pm 0.0005$. This is seen by many cosmologists as support for an idea like cosmic inflation for the origin of the perturbations.

It may be interesting to note the coincidence that $n = H_0 t_0$. In fact $n/(H_0 t_0) = 1.004 \pm 0.007$.

Another way to describe perturbations focuses on how they are growing today. In the Λ CDM model, this is strongly affected by the presence of a cosmological constant, which impedes the amplification of structure at relatively recent times. Relative to a flat model with vanishing Λ , the 'growth suppression factor' is $g = 0.784 \pm 0.006$.

Curvature

Although we do not know if the whole extent of space is finite or infinite, we can measure curvature within our Hubble patch (i.e. volume of our observable part of the Universe). *Planck* (together with other data sets, see [3]) yields $\Omega_k = -0.000 \pm$ 0.003, where $\Omega_k = 1-\Omega_{tot}$. This means that the total density (in matter plus radiation plus dark energy) is quite accurately given by ρ_{crit} .

Constraints can be placed on the radius of curvature, such that $R_{\rm curv}/R_{\rm H} > 12$ (at 95% confidence, with $R_{\rm H} \equiv c/H_0$). The particle horizon (i.e., maximum distance that particles can have travelled in the age of the Universe since the end of a possible inflationary period) is also well-defined, $R_{\rm p} = Xc/H_0$, with $X = 3.21 \pm 0.04$. For particles starting at the last-scattering surface (before which the Universe is optically thick to CMB photons), we find $X = 3.15 \pm 0.04$. Using this to define an observable volume and considering constraints on curvature, we can derive a lower limit to the number of such volumes in the *entire* Universe (assuming that our own patch is a fair sample of course): $N_{\rm U} > 250^{[24]}$.

Observable Universe

We cannot say whether there are an infinite number of particles in the entire Universe. However, we can determine the number in the *observable* Universe, which has a finite volume. Using the above definition of the observable distance (as the distance to the last-scattering surface), and assuming flat geometry, we find that the radius of the observable Universe is (429.2 \pm 1.3) Ym (with the particle horizon being only about 2% larger, (437.9 \pm 1.3) Ym). Here the prefix 'Y' is for 'Yotta', meaning 10^{24} , the largest approved SI unit multiplier. It may be a coincidence that, for the sizes of *anything* observable in metres, we do not need a larger prefix.

The total number of baryons contained within the observable Universe is then $N_{\rm b} = (8.27 \pm 0.11) \times 10^{79}$. For photons we have $N_{\gamma} = (1.360 \pm 0.012) \times 10^{89}$, and the total number of known particles (dominated by photons and massless neutrinos) is $N_{\gamma+\nu} = (2.49 \pm 0.02) \times 10^{89}$ ($\sim \alpha^{-42}$).

Acoustic scales

The CMB variations are largely determined by oscillating sound waves, with a wide range of wavelengths. Because of the finite speed of propagation of these acoustic modes, and the finite age of the Universe, a characteristic scale is built in by the physics. At the distance of the last-scattering surface this length scale projects onto a particular angular scale, which is effectively the angular size of 'blobs' in CMB maps. In conventional units, this scale is $\theta_* = 0.5968^\circ \pm 0.0003^\circ \simeq 0.6^\circ$. This is essentially the same as (only about 10% larger than) the angular diameter of the Sun and the Moon.

Rescattering

A fraction of the CMB photons are scattered in a period of relatively recent reionisation of the Universe. This is often expressed as an optical depth, but more directly, the rescattered fraction is about 8.8%. The distance out to which the Universe is ionised, i.e., the distance to the reionisation surface, is (305 \pm 6)Ym.

Planck units

The quantities that describe the Universe could be given in different systems of units. The system of 'Planck units' is formed by using the speed of light (c), reduced Planck constant (\hbar), and gravitational constant (G) to form the Planck length ($l_{\rm p} = \sqrt{\hbar G/c^3}$), Planck time ($t_{\rm p} = \sqrt{\hbar G/c^5}$), Planck mass ($m_{\rm p} = \sqrt{\hbar c/G}$), and Planck temperature ($T_{\rm p} = \sqrt{\hbar c^5/Gk^2}$). In these units, we have: $t_0 = (8.08 \pm 0.02) \times 10^{60} t_{\rm p} (\simeq 5 \times 2^{200} t_{\rm p})$; $H_0 = (1.185 \pm 0.013) \times 10^{-61} t_{\rm p}^{-1}$; $\Lambda = (2.91 \pm 0.12) \times 10^{-122} t_{\rm p}^{-2}$; and the CMB tem-

perature today $T_0 \simeq (1/41)2^{-100} T_{\rm p}$, or $T_0/T_{\rm P} = (160/3^8)2^{-100}$. So to use an analogy with the musical scale, one can say that the CMB temperature today is one hundred octaves, eight perfect fifths, and one justly tuned minor fifth below the Planck temperature.

The particle content of the Universe is related to the total entropy. One can define the asymptotic 'Gibbons-Hawking entropy' ^[25] for de Sitter space as 1/4 the asymptotic cosmological horizon area in Planck units, i.e., $S/k \equiv 3\pi/(\Lambda t_{\rm P}^2) \simeq 5(t_0/t_{\rm P})^2$. This is $(3.24 \pm 0.12) \times 10^{123} \simeq 5^3 2^{400}$.

MNEMONIC COSMOLOGY

Martin Rees wrote a popular cosmology book entitled 'Just Six Numbers'^[26]. Although his numbers differ from the six which are well measured in today's cosmological data, the basic message is the same: we have developed an understanding of the large-scale Universe that is rather simple, is described by roughly a handful of numbers, and would be quite different if the numbers were different. The Standard Model of Cosmology is built on a framework of assumptions which are reasonable and few in number. Within that framework only half a dozen parameters are required to fit the current data. However, we have several choices for how to present these numbers, including the epoch at which to specify them, the units to use, and whether to focus on dimensionless ratios. Since these

are the quantities which describe the entire observable cosmos, then it is worth manipulating and evaluating them, in order to better grasp how our Universe measures up.

Lots of different numbers have been presented here, with the expectation that distinct choices might appeal to different people. In Table 3 we have gathered together some of our favourite numerical facts about the whole observable Universe.

ACKNOWLEDGEMENTS

This research was supported by the Natural Sciences and Engineering Research Council of Canada and the Canadian Space Agency. We thank Hilary Feldman for useful comments on an earlier draft, and members of the Planck Collaboration, particularly Jim Zibin, for helpful discussions.

TABLE 3A selection of numbers that describe our Universe

Cosmic Mnemonics				
Symbol	Quantity	Value		
t_0	Age of the Universe today	$\simeq 5$ trillions days $\simeq 5 \times 2^{200} t_{\rm p}$		
Λ	Cosmological constant	$= 10^{-35} \text{ s}^{-2} = \text{ten square attohertz}$		
$H_0 t_0$	Expansion rate times age today	$\simeq 0.96 = n$		
H_∞	Future limit for Hubble parameter	$\simeq 56 \mathrm{km}\mathrm{s}^{-1}\mathrm{Mpc}^{-1} = \sqrt{10/3}\mathrm{attohertz}$		
Z_q	Redshift at which acceleration was zero	~ 0.65		
z_{\oplus}	Redshift of formation of the Earth	= 0.42		
θ_*	Characteristic scale of CMB anisotropies	$\simeq 0.6^{\circ} \sim$ solar angular diameter		
Ω_{γ}	Density parameter for photons	$= \alpha^2$		
η_{10}	Baryon-to-photon ratio $(\times 10^{10})$	$\simeq 2\pi$		
$R_{\rm obs}$	Radius of the observable Universe	$\sim 400 \text{ Ym}$		
$N_{\rm part}$	Number of particles in observable Universe	$= \text{few} \times 10^{89} \sim \alpha^{-42}$		
$R_{\sigma=1}$	Scale for density contrast of unity	$\simeq 9 \text{ Mpc}$		
$\sigma_{ m M}$	Hubble-scale perturbation at Milne epoch	$\simeq 6 \times 10^{-6}$		

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